Analytic potentials for the forward and inverse modeling of SP anomalies caused by subsurface fluid flow

Pascal Sailhac and Guy Marquis
IPGS, UMR 7516 CNRS-ULP, Strasbourg, France

Abstract. Classical interpretation techniques developed primarily for gravity and magnetic data have been adapted to electric self-potential (SP) data. From a recently developed wavelet-based technique for potential field interpretation, we propose a novel approach for the forward and inverse modeling of SP anomalies caused by subsurface fluid flow. We use analytic signals and wavelets associated with the electric and fluid flow potentials. Fluid flow singularities have typical signatures in the wavelet-domain SP data that can be used in the inversion scheme. For cases where electrokinetics is unambiguously the origin of SP anomalies, our interpretation method provides key parameters of the subsurface fluid flow (e.g., position, geometry, flow rate). To illustrate the theory, we consider synthetic 2D flows for which analytic solutions exist and actual SP data acquired around the Vulcarolo fissure on Mount Etna that show sources between 3 to 18 m depths.

Introduction

For years, geophysicists have studied temporal and spatial variations of electric self-potential (SP) for mineral exploration, for monitoring of hydrothermal or subsurface fluid flow, or as possible earthquake precursors (e.g., Yoshida et al. [1997] and references therein). This paper deals with surface SP anomalies resulting from electrokinetic phenomena (see [Revil et al., 1999] and references therein). Subsurface flow characterization based on SP data classically relies on the search for equivalent electric sources, that could be interpreted in terms of fluid flow under assumptions of a homogeneous host medium and some coupling coefficient [Sill, 1983]. This inverse problem of surface electric SP can be approached by using techniques designed for inverting other potential fields. We consider a wavelet transform-based technique to determine geometric parameters for each source of anomaly [Moreau et al., 1999; Sailhac et al., 2000]. This is based on the use of Cauchy or Poisson wavelets that allow simple inverse schemes in the wavelet domain of potential fields. In addition, this method has higher lateral and depth resolutions than those based on statistics in the Fourier domain. By applying this continuous wavelet transform, SP data are replaced by the set of their analytic signals upward continued at a series of altitudes. This transform is similar to that used in other techniques based on derivatives or analytic signals of SP data [Abdelrahman et al., 1998; Sundararajan et al., 1998]. The upward continuation process results in noise reduction with a scale parameter related to the altitude of continuation and the source depth. Moreover, this continuous wavelet transform is also a set of correlation coefficients between the data and the Green’s functions from various source locations; it can be related to the space of probability introduced for SP tomography by Patella [1997]. In this paper, we assume a homogeneous medium (for streaming coupling coefficient and electrical conductivity) and illustrate our method with synthetic cases of 2D flow (uniform flow, pipes and walls of sources and sinks). We illustrate our technique by an application to SP data acquired across the Vulcarolo fissure on Mount Etna [Aubert, 1999].

A new technique for SP modeling

The first novelty of our technique is to apply the inverse scheme in the wavelet domain. Inversion in the wavelet domain provides estimates of both the location and nature (e.g., dipole, etc.) of the electric current sources causing SP anomalies [Gibert and Pessel, 2001].

The second novelty is to include fluid flow potential functions in the inverse scheme: hence modeling is not reduced to the determination of equivalent electric sources (as is the case for classical methods) but to parameters of the underground fluid flow velocity field. The simplest cases involve stream flow with homogeneous discharge functions directly proportional to the wavelet coefficients of the observed SP anomaly.

SP modeling

For a 2D flow, the fluid flux $\mathbf{u}$ (in m.s$^{-1}$) is derived from a potential function $\varphi$ (in m$^2$.s$^{-1}$) such that $\mathbf{u} = -\nabla \varphi$ (e.g., [Bear, 1988]).

Consider a vertical section in which $x$ is the abscissa along the profile and $a > 0$ is the altitude (or depth when $a < 0$). Coupling coefficients between the stream flow potential $\varphi$ and electric potential $\phi$ (in V) can be derived from Onsager relations and the conservation of electric flux [Revil et al., 1999]:

$$\nabla (\sigma \mathbf{E}) = -\nabla (\mu_f k^{-1} \mathbf{u}),$$

(1)

where $\mu_f$ is the dynamic shear viscosity of the pore fluid (in Pa.s); $l$ is the electrokinetic coupling tensor (in m$^2$.V$^{-1}$.s$^{-1}$); $\sigma$ and $k$ are the electrical conductivity (in $\Omega^{-1}$.m$^{-1}$) and the intrinsic permeability (in m$^2$) tensors of the porous medium respectively; $\mathbf{E}$ is the electric field (in V.m$^{-1}$).

For a homogeneous medium (where $\nabla \sigma = 0$), the simplified equation of conservation (1) relates the electric potential $\phi$ to the water flux $\mathbf{u}$:

$$\nabla^2 \phi = -\nabla (\eta \mathbf{u}),$$

(2)
where \( \eta = \sigma^{-1}\mu_f k^{-1} \) is the voltage coupling tensor (in \( \text{V.m}^{-1}/(\text{m.s}^{-1}) \)).

The second term of Poisson’s equation (2) is the source term equivalent to an electric charge of \( \varepsilon \nabla (\eta \mathbf{u}) \) where \( \varepsilon \) is the dielectric permittivity of the medium. For simplicity, let us assume a homogeneous isotropic medium of constant scalar voltage coupling \( \eta \); Poisson’s equation becomes:

\[
\nabla^2 \phi = \eta \nabla^2 \varphi. \tag{3}
\]

This form of Poisson’s equation (3) shows the correspondence between electric sources (for \( \nabla^2 \phi \)) and flow singularities (for \( \nabla^2 \varphi \)). Thus, when boundary conditions of both potentials \( \phi \) and \( \varphi \) are approximately the same, one can analyze them simultaneously using simply \( \phi = \eta \varphi \).

The flow potential \( \varphi \) is also related to the pore fluid overpressure \( \delta P \) (in Pa) and density perturbation \( \delta \rho_f \) (in kg.m\(^{-3}\)) by Darcy’s law:

\[
\mathbf{u} = -k\mu_f^{-1}(\nabla \delta P - \delta \rho_f \mathbf{g}), \tag{4}
\]

where \( \mathbf{g} \) is the gravity field (in m.s\(^{-2}\)).

Let us now consider the stream function \( \psi(x,a) \) associated to the flow potential \( \varphi(x,a) \) by a Hilbert transform \( \mathcal{H} \); its isovales indicate the streamlines. We can define the stream complex potential as a function of the complex variable \( \zeta = x + ia \):

\[
\tilde{\psi} = f(\zeta) = (1 + i\mathcal{H})\varphi(x,a) = \varphi(x,a) + i\psi(x,a). \tag{5}
\]

The horizontal and vertical velocity components are given by the real and imaginary parts of the derivatives \( f'(\zeta) \) along \( \zeta \): \( u_h = -Re[f'(\zeta)] \) and \( w_v = Im[f'(\zeta)] \). It is the opposite to the discharge potential \( \Omega(\zeta) = -f'(\zeta) = u_h - iw_v \).

The complex wavelet coefficients are rescaled derivatives of the complex electric potential \( \tilde{\varphi}(\zeta) = (1 + i\mathcal{H})\varphi(x,a) \) calculated from the electric potential \( \varphi \) and upward continued at an altitude \( a \) above the measurement surface. With a derivative of order \( \gamma \), this is defined by [Sailhac et al., 2000]:

\[
W^\gamma_c(x,a) = (a \frac{d}{d\zeta})^\gamma \tilde{\varphi}(\zeta)|_{\zeta=x+ia}. \tag{6}
\]

With constant coupling parameters, they can be related to the stream complex potentials:

\[
W^\gamma_c(x,a) = a\eta f'(\zeta); \quad \text{and} \quad W^\gamma_c(x,a) = a^2\eta f''(\zeta). \tag{7}
\]

### Synthetic flows

Using Equations (7), several subsurface flow geometries can be analyzed analytically from the wavelet coefficients of SP data and models of stream function.

For a uniform flow in a vertical section with a dip angle \( \alpha \) and a velocity \( u_0 \), the flow can be described from the first-order wavelet coefficient \( W^1_c \): \( f'(\zeta) = -u_0e^{-i\alpha} \). The modulus is a constant proportional to the velocity and the phase is its direction.

The second-order wavelet coefficient \( W^2_c \) of this uniform flow vanishes: this property can be useful for separating a singular flow having non-zero second-order wavelet coefficients from unknown uniform flow. In addition, applying the second-order wavelet coefficient removes the effect of the gravity field that contributes a constant vertical component in the flow (term \( k\mu_f^{-1}\delta \rho_f g \) in Darcy’s law (4)).

For a sink or pump of intensity \( m \) (in \( \text{m}^3.\text{s}^{-1} \)) at \( \zeta_0 = x_0 - iz_0 \), \( x_0 \), \( z_0 \) and \( m \) can be given from \( W^1_c \) or \( W^2_c \): \( f'(\zeta) = m(\zeta - \zeta_0)^{-1} \) and \( f''(\zeta) = -m(\zeta - \zeta_0)^{-2} \).

For a corner flow near the border of a \( \pi/\alpha \)-dipping ramp, the intensity \( m' \) (in \( \text{m}^3.\text{s}^{-1} \)), the location of the corner (parameter \( \zeta_0 = x_0 - iz_0 \)) and its angle (\( \alpha \)) can be determined by the inversion of wavelet coefficients: \( W^2_c(x,a) = a^2\eta(\alpha - 1)m'(\zeta - \zeta_0)^{\alpha-2} \).

This inversion can be performed by picking the maximum wavelet coefficient modulus for each altitude and applying the scheme introduced by Moreau et al. [1999]: they use a linear regression of the plots of \( \log |W^2_c|/a^2 \) versus \( \log(a + z_0) \) where \( z_0 \) is an a priori depth. The misfit of a least-squares fitting gives the depth of the anomaly.

**Figure 2.** Log-log plots of the normalized wavelet coefficients versus the scale \( a + z_0 \) for selected complex synthetic flows having two scale parameters, the depth \( z_0 = 1 \) and a distance \( d = 2 \): a source-sink couple (a), a two-source couple (b), and a horizontal finite-wall source (c). The type of flow is given by the slope in log-log plot: \(-2 \) for sources or \(-3 \) for a source-sink couple. \( d \) is obtained from the limit of \( H(a) \).
regression allows one to discriminate between a series of \( z_0 \) values (see Fig. 1). The slope corresponds to an exponent related to the geometry (e.g. the dip of a ramp).

For a flow with two distance parameters (some characteristic length \( d \) and the depth \( z_0 \)), linear regression is only the first step of the inversion. The fit can be improved by adjusting a specific scale-dependent residual function introduced by a Taylor expansion. We follow Sailhac et al. (2000) who obtained an estimate for magnetic source thickness by using the limit of the function \( H(a) = 2(z_0 + a)/\sqrt{R(a)} \) where \( R(a) \) is the linear regression residual.

For a source-sink couple (Fig. 2a), with the source at \((x_0 + d, z_0)\) and the sink at \((x_0 - d, z_0)\), with \( m > 0 \) in \( \text{m}^2 \text{s}^{-1} \), we find \( f(\zeta) = m \log[(\zeta - \zeta_0 + d)/(\zeta - \zeta_0 - d)] \), and \( f''(\zeta) = -4dm(\zeta - \zeta_0)^{-3}[1 - d^2/(\zeta - \zeta_0)^2]^{-2} \), where \( \zeta_0 = x_0 - iz_0 \). At large scales, the wavelet modulus maxima are given by \( |W_0^2(a)| \approx 4dm|a|^2(z_0 + a)^{-3}|1 - 2d^2/(z_0 + a)^2| \). Thus the limit of \( H(a) \) equals \( 2\sqrt{3}d \).

For a flow from two sources (Fig. 2b) at \((x_0 \pm d, z_0)\) with intensity \( m \), one finds \( f(\zeta) = m \log[(\zeta - \zeta_0)^2 + d^2] \) where \( \zeta_0 = x_0 - iz_0 \). The limit of \( H(a) \) equals \( 2\sqrt{3}d \).

For a flow from a horizontal finite-wall source (Fig. 2c) at depth \( z_0 \) between \( x_0 - d \) and \( x_0 + d \), with a uniform density \( m/2d \) in \( \text{m} \text{s}^{-1} \), one finds \( f(\zeta) = m/2d \int_{-d}^{d} \log(\zeta - \zeta_0 - z)dz \) where \( \zeta_0 = x_0 - iz_0 \). The limit of \( H(a) \) therefore equals \( 2d \).

Application to the Vulcarolo fissure

Hydrothermal circulation near volcanoes is often monitored to characterize possible volcanic crises, and SP is one of the preferred geophysical methods for this task (e.g. Antraygues and Aubert, 1993). As an illustration of the applicability of our inversion method, we consider here surface SP data acquired by Aubert [1999] across the Vulcarolo fissure, about 600 m south of the center of Mt Etna.

The SP data have been acquired while the craters were in a steady state. Aubert [1999] showed that the fissure was the locus of active convection. We have calculated the wavelet transform of the data across a profile crossing the fissure (profile ABC in Aubert [1999] where a map can also be found). They exhibit a region of modulus maxima when crossing the fissure (Fig. 3).

The wavelet coefficients along the line of modulus maxima have been inverted (Fig. 4) by fitting a scaling relation similar to that previously shown on synthetics: \( \log(|W_0^2(x, a)/a^2|) \sim \log(I) + \beta \log(a + z_0) \) where \( a \) is the altitude, \( z_0 \) is the depth to the source, \( I \) is an intensity factor, and \( \beta \) a shape exponent.

Our new inversion technique localized the hydrothermal circulation in the fissure between 3 and 18 m depths. We can then relate this information to fluid flow parameters. Using a dynamic viscosity of \( \mu_f = 8.79 \times 10^{-4} \text{ kg/m/s} \), a coupling parameter of \( C = 1/\sigma \sim \text{100mV/MPa} \) and a permeability of \( k \sim 10^{-12} \text{ m}^2 \) [Jouniaux et al., 2000], the electric intensity \( I = 2400 \text{ mV/m} \) yields the flow intensity \( m' \sim 0.01 \text{ m}^3/\text{s} \). By comparison with the synthetic examples, the shape exponent \( \beta = -3 \) suggests circulation around a semi-infinite horizontal plane (ramp with \( a = -1 \)) or a source-sink couple.

With regard to the hydrothermal activity, we suggest that the latter is more realistic and would correspond to an active fissure of about 15 m height and 0.1m/s strength. Further analysis of temperature and SP depth soundings together with surface SP surveys will improve this model.

Conclusions

Our technique is meant as a fast interpretation tool yielding a starting model for more elaborate modeling, and not as a final modeling step. Indeed, its limitations arise from its two restrictive assumptions:

i. Equation (3) neglects the effects of secondary electric sources caused by a non-homogeneous resistivity and additional distortions of surface SP anomalies due to coupling coefficient variation.

ii. The proportionality between hydraulic and electric potentials requires the same boundary conditions for both potentials in the application of relation (3); this is not generally the case. To consider cases with distinct boundary conditions, the development of different analyzing functions is necessary.

Acknowledgments. Thanks to M. Aubert (OPGC, Clermont-Ferrand) for providing the data from Vulcarolo and to two anonymous referees for pointing out several shortcomings in the first version of this paper.
References


G. Marquis and P. Sailhac, Proche Surface, IPGS, 5 rue René Descartes, Strasbourg 67084, France. (Guy.Marquis, Pascal.Sailhac@eost.u-strasbg.fr)

(Received October 6, 2000; revised February 5, 2001; accepted February 9, 2001.)