Estimation of hydraulic conductivity of an unconfined aquifer using cokriging of GPR and hydrostratigraphic data

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Abstract

Densely sampled geophysical data can supplement hydrogeological data for estimating the spatial distribution of porosity and hydraulic conductivity over an aquifer. A 3D Ground Penetrating Radar (GPR) survey was performed over a shallow unconfined aquifer consisting of a coarse to medium sand sequence overlying an impermeable clay layer. The site is instrumented with piezometers and water levels are frequently monitored. Vertical determination of moisture and granulometry at a resolution of 10 cm were made at a few locations. The GPR reflection times were correlated with piezometric and stratigraphic information; cokriging of both data yields the spatial distribution of the radar velocities within the layers. Porosity and hydraulic conductivities are estimated using the Complex Refractive Index Method (CRIM) and Kozeny–Carman formulations, respectively. A pumping test and a tracer test, both done using a well in the center of the survey zone, provide a measure of the average hydraulic conductivity and its anisotropy. The results from cokriging in the saturated zone show that the estimated parameters agree very well with the measured hydrogeological data. The geometric mean of the porosity is close to the laboratory measurements. The geometric mean of the GPR-derived hydraulic conductivities fits the values obtained from the pumping and tracer tests. The range of estimated hydraulic conductivities is quite large and indicates that flow could be faster or slower than the one predicted from the pumping test in some places. Radar attenuation is also found to be a good indicator of porosity distribution. From the observed (high) GPR attenuations and electrical conductivities of water sampled in the piezometers, porosity is determined using Archie’s formula. In the vadose zone, moisture content estimated from the GPR velocities using either CRIM or Topp formulations agree well with the ones from the laboratory measurements. Cokriging of the radar reflection times and of the hydrogeological/stratigraphic data leads to an accurate estimate of the radar velocities with a precision and a spatial resolution much higher than the CDP technique. Within the limits of the interpretative models, porosity, saturation and hydraulic conductivities can accurately be estimated with a high spatial resolution over the survey zone. © 2001 Published by Elsevier Science B.V.

Keywords: GPR; Cokriging; Hydraulic conductivity; Porosity; Water content; Dielectric constant

1. Introduction

Knowledge of the distribution of porosity and hydraulic conductivity is of paramount importance in...
environmental sciences because these parameters control water and pollutants movement. Usually in hydrogeological studies, porosity is estimated by the ratio of the bulk mass density of core samples and the particle mass density and hydraulic conductivities are estimated by a pumping test and/or by the Kozeny–Carman relation (Freeze and Cherry, 1979, p. 337–352). Unfortunately, the number of in situ hydraulic data is generally not sufficient to have an accurate determination of the spatial field of conductivities. Moreover, a pumping test gives only a weighted average of the horizontal hydraulic conductivity at a scale of several meters (Acevedo, 1996).

The usual way to model the hydraulic conductivity field in more detail consists in constraining the field with additional information such as hydraulic head measurements during a pumping test (Chapuis et al., 1998). The abundance of indirect measurements of conductivities, even if inaccurate, should allow a better modeling of the conductivity field, and consequently, of the flow (Young, 1996).

Geophysical methods provide, at a very low cost, a large amount of information on various physical properties and on the stratigraphy of the soil (van Overmeen, 1994a,b; Knight, 1997). The problem of integrating hydro-stratigraphical and geophysical data measured at various scales can be resolved using geostatistics. For example, Knight (1997) and Rea and Knight (1995) used Ground Penetrating Radar (GPR) data to detail the stratigraphy of soils. The structure of the well-sampled geophysical data can be used to interpolate incompletely sampled hydrogeological data on a fine and regular grid. GPR allows the determination of the EM travel time from the surface to an interface between two layers of different dielectric constants. GPR velocity and impedance changes are strongly dependent on water content. Hence, a strong correlation exists between GPR data and hydrogeological parameters. The summary of estimated parameters used in this study is shown in Table 1.

A test survey was performed on a small grid of $6 \times 5$ m at the test site of Ecole Polytechnique, located in Lachenaie, Quebec (Canada). This test site consists in a 3.5-m medium-to-coarse sand layer partially saturated resting on a 20-m clay layer. Hydraulic head measurements were obtained from seven piezometers penetrating to the depth of the clay layer. The 2D distribution of the saturated and unsaturated layer thicknesses was determined combining piezometric, stratigraphic and GPR data with geostatistics. Once the depths to the piezometric level and to clay are determined, the travel times are used to compute the velocity field. The velocity itself can be related to the volumetric water content using the CRIM relation (Dannowski and Yamaranci, 1999; Garrouch and Sharma, 1994; Hubbard, 1997; Knight, 1997; Knight and Abad, 1995; Knight and Endres, 1990; Knight and Nur, 1987; Knoll et al., 1995). In addition, the Kozeny–Carman relation (Todd, 1980, p. 361; Chapuis and Montour, 1992) is used to estimate the hydraulic conductivity. Topp’s formulation can be also used to compute the water content from the dielectric constant (Topp et al., 1980).

The bulk electrical conductivity can be estimated from the observed radar attenuations in the unsaturated and saturated sands. Using Archie’s formula, the porosity can be computed given the electrical conductivity of the water sampled in the piezometers. The porosity estimated from attenuation coeffi-

<table>
<thead>
<tr>
<th>Properties</th>
<th>Saturated zone</th>
<th>Unsaturated zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi$ GPR (%)</td>
<td>CRIM</td>
<td>$37 \pm 0.05%$</td>
</tr>
<tr>
<td></td>
<td>Topp</td>
<td>$37 \pm 0.05%$</td>
</tr>
<tr>
<td></td>
<td>DC</td>
<td>36%</td>
</tr>
<tr>
<td>$\Phi$ Laboratory (%)</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>$K$ GPR (m s$^{-1}$)</td>
<td>all piezo</td>
<td>$8.0 \times 10^{-4} \pm 2.2 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>three piezo</td>
<td>$7.8 \times 10^{-4} \pm 1.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>$K$ pumping test (m s$^{-1}$)</td>
<td>three piezo</td>
<td>$7.8 \times 10^{-4} \pm 1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>$V_e$ GPR (%)</td>
<td>X</td>
<td>$22 \pm 4%$</td>
</tr>
<tr>
<td>$V_e$ Laboratory (%)</td>
<td>X</td>
<td>15%</td>
</tr>
</tbody>
</table>
The theoretical overview

2.1. Geostatistics

This section shows how the spatially well-sampled GPR reflections can be used to estimate the spatial distribution of hydraulic head and depth-to-clay on a fine regular grid. To do this, we introduce three concepts: cokriging, covariography and intrinsic correlation.

Cokriging (Journel and Huijbregts, 1978; Myers, 1982) is a mathematical interpolation and extrapolation tool which uses the spatial statistics between a secondary variable (here, the two-way travel times to radar reflectors) and a primary variable (here, depths to water table and sand–clay interface) to improve estimation of the primary variable at unsampled locations.

The estimator is a linear combination of weights, \( \lambda \) and \( \tau \), with data from two variables located at sample points in the neighborhood of point \( x_i \).

\[
d_{i}^{*} = \sum \lambda_{oi}d_{i} + \sum \tau_{oj}t_{j} \quad \text{(1a)}
\]

\[
\sum \lambda_{oi} = 1 \quad \text{and} \quad \sum \tau_{oj} = 0 \quad \text{(1b)}
\]

for the non-bias conditions.

\( d_{i}^{*} \) is the depth estimated at \( x_i \), \( d_{i} \) is the depth to interface at position \( x_i \) and \( t_{j} \) is the two-way travel time to interface at \( x_j \).

The cokriging consisting of minimizing the variance of estimation error, given by

\[
\sigma^{2} = \mathbb{E}\left[(d_{i}^{*} - \sum \lambda_{oi}d_{i} - \sum \tau_{oj}t_{j})^{2}\right] \quad \text{(2)}
\]

subject to the constraints on the sum of the weight \( \lambda_{oi} \) and \( \tau_{oj} \) for each location where the variance is to be estimated, involving the calculation of the covariances and cross-covariances. The covariance can be estimated by the variogram described below. The Lagrange multiplier method is used to minimize the function subject to its two constraints to give the variable estimate at any \( x_i \).

The benefit of cokriging in geophysics has already been demonstrated in the use of seismic refraction data for imaging bedrock topography (Chiles and Delfiner, 1999, p. 302; Nyquist et al., 1996) and in gas reservoir modelling (Journel and Huijbregts, 1978, p. 335).

A useful tool to verify the performance of the cokriging is cross-validation. It consists in cokriging the measured values after removal of data one at a time for each variable. If the difference between measured and calculated data is small, then, the cokriging performs well. Also, the standardized errors of prediction should present a variance close to one. If this is not the case, then, the coregionalization model should be revised.

The covariogram, \( C(h) \), (or covariance or autocorrelation function) measures the covariance of the variable of interest at two points as a function of the distance between these points. This function is usually decreasing with increasing distance and for a critical distance, the range becomes null. At distance of zero, the covariogram value is simply the variance of the variable of interest (Journel and Huijbregts, 1978, pp. 303–444; Chiles and Delfiner, 1999, pp. 292–374). The covariogram is defined as (Wackernagel, 1995, p. 37)

\[
\begin{align*}
\mathbb{E}[Z(x)] &= m \\
\mathbb{E}[Z(x)Z(x+h)] - m^2 &= C(h) \\
\text{for all } x, x+h \in D
\end{align*}
\]

where \( \mathbb{E}[Z(x)] \) is the mathematical expectation of the random function \( Z(x) \) (here, the depth or the reflection time). The mean of each variable \( Z(x) \) within the domain is equal to a constant \( m \), \( Z(x) \) is the value of the function at the position \( x \), \( Z(x+h) \) is the value of the function at the position \( x+h \) and \( D \) is the spatial domain.
The cross-covariogram (also called cross-covariance or cross-correlation function) measures the covariances between two different variables as a function of the distance vector between the points where these variables are defined. The cross-covariogram, \( C_{ij}(h) \) of a set of two random functions \( Z_1(x) \) and \( Z_2(x) \) is defined as (Wackernagel, 1995, p. 131)

\[
\begin{align*}
E[Z_i(x)] &= m_i, \quad i = 1, 2 \\
E[(Z_i(x) - m_i)(Z_j(x + h) - m_j)] &= C_{ij}(h) \\
&\text{for all } x, x + h \in D
\end{align*}
\]

(4)

where the mean of each variable \( Z_i(x) \) at any point of the domain is equal to a constant \( m_i \).

A particular model, called intrinsic correlation, arises when the covariances and cross-covariances of two variables can be assumed proportional to each other. The coefficient of proportionality between the two covariograms is then equal to the ratio of the variances and the simple correlation between the two variables suffices to define the cross-covariogram (Chiles and Delfiner, 1999, pp. 336–339; Journel and Huijbregts, 1978, p. 326).

2.2. Ground penetrating radar principles

A good tutorial of the GPR theory and practice can be found in Ulriksen (1982) and Davis and Annan (1989). Here, we briefly present the important concepts.

If a very short EM pulse is transmitted by an electric dipole into the ground, it propagates in the subsurface with a velocity depending on the electrical properties of the ground. For a layered subsurface with contrasting electrical properties, a part of the EM energy, is reflected back to the surface where it is detected by a receiver dipole and recorded. Synchronization between the transmitter and the receiver systems allows the determination of the time taken for the EM pulse to be reflected back. Usually, the EM pulse is defined by a central frequency and a bandwidth. The radar frequency range is commonly 10–1000 MHz. For a monochromatic radar wave in the far field domain, the amplitude of the EM wave, \( A_i \), at a distance \( r \) of the source is given by the initial EM pulse amplitude, \( A_0 \), the attenuation constant, \( \alpha \) and the phase constant, \( \beta \).

\[
A_i = \frac{A_0}{r} e^{-\alpha \gamma \beta r} e^{-i\omega t}
\]

(5)

with

\[
\begin{align*}
\alpha &= \omega \sqrt{\mu \varepsilon} \left[ \frac{1}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} - 1 \right) \right]^\frac{1}{2} \\
\beta &= \omega \sqrt{\mu \varepsilon} \left[ \frac{1}{2} \left( \sqrt{1 + \left( \frac{\sigma}{\omega \varepsilon} \right)^2} + 1 \right) \right]^\frac{1}{2}
\end{align*}
\]

(6)

where \( \omega \) is the angular frequency, \( \mu \) is the magnetic permeability (H/m), \( \varepsilon \) is the electrical permittivity (F/m) and \( \sigma \) is the electric conductivity (S/m) and \( i = (-1)^{1/2} \). The attenuation constant characterizes the amplitude reduction of the radar wave caused by the physical properties of the transmitting media.

From \( \beta \), the phase velocity of the radar signal can be determined

\[
v = \frac{\omega}{\beta}
\]

(7)

For low-loss medium (good dielectric), \( \left( \sigma/\omega \varepsilon \right)^2 \ll 1 \) (Balanis, 1989, p. 149; Annan and Daniels, 1998), \( v \) becomes

\[
v = \frac{\omega}{\beta} = \sqrt{\frac{1}{k \mu_0 \varepsilon_0}}
\]

(8)

where \( k = \varepsilon / \varepsilon_0 \) is the relative permittivity or dielectric constant, \( \varepsilon \) and \( \varepsilon_0 \) are the dielectric permittivities of the medium and of the free space, respectively. Usually, the magnetic properties of most geologic materials do not vary significantly from the magnetic properties of free space. Therefore, the effect of variations in magnetic properties do not have to be considered when making electromagnetic measurements and \( \mu = \mu_0 \), permeability of the free space \( (4\pi \times 10^{-7} \text{ H/m}^{-1}) \).

At an interface between two layers of contrasting electrical properties, the radar wave is partitioned into a reflected and a transmitted wave. The reflected
amplitude depends on the reflection coefficient, $R$, which is, for normal incident signal, given by

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (9)$$

where, $Z_{j(1,2)}$ is the electrical impedance of the medium $j$.

$$Z_j = \sqrt{\frac{i \omega \mu_j}{\sigma_j + i \omega \varepsilon_j}} \quad (10)$$

For low-loss medium, i.e. $(\sigma / \varepsilon) \ll 1$, $R$ is equal to

$$R = \frac{\sqrt{k_1} - \sqrt{k_2}}{\sqrt{k_1} + \sqrt{k_2}} \quad (11)$$

where, $k_{j(1,2)}$ is the dielectric constant of the medium $j$.

As noted by Annan and Daniels (1998), generally, the dielectric constant is complex

$$k = k' + i \left( k'' + \frac{\sigma_{dc}}{\omega \varepsilon_0} \right) \quad (12)$$

where, $k$ is the complex dielectric constant of the soil, $k'$ is the real part of the dielectric constant, $k''$ is the imaginary part or the electric loss and $\sigma_{dc}$ is the zero-frequency conductivity. The variables, which affect the electrical response in soils, are texture, structure, soluble salts, water content, temperature, density, and measurement frequency. Over the frequency range of 10 MHz–1 GHz, the real part of the dielectric constant does not appear strongly frequency dependent (Todd, 1980). Davis and Annan (1989) indicate that the dielectric loss, $k''$, is considerably less than $k'$ in this frequency range. Thus, when operating at radar frequencies and if the soil is a good dielectric, the dependence of radar velocity on electrical conductivity is negligible. In such medium, the velocity of EM waves is related only to the real part of the dielectric constant (Eq. (8)).

Even though not easily measurable, the effects of electrical loss may exist. Therefore, Topp et al. (1980) call the measured dielectric constant apparent dielectric constant, $k_a$. For low loss and nearly homogeneous materials, $k_a = k' = k$.

Knowing the depth of the reflectors (from stratigraphic control) and reflection times allow the determination of the horizontal distribution of velocity in each layer. Once the velocity distribution is known, the dielectric constant distribution is computed using Eq. (8).

### 2.3. Determination of porosity and hydraulic conductivity

A relation between dielectric constant and porosity, extensively described by Knoll et al. (1995), Hubbard et al. (1997), Knight (1997) and Yu et al. (1999) is modeled by the Complex Refractive Index Method (CRIM). This relation was first used in seismic for oil reservoir estimation, and after some modifications, was extended to describe the total dielectric constant (Wharton et al., 1980)

$$\sqrt{k'} = \sum_i V_i \sqrt{k_i'} \quad (13)$$

where, $V_i$, volume fraction of the component $i$, $k_i'$ is the complex dielectric constant of the component $i$.

This relation can be expanded for the case of unsaturated medium. If the matrix is assumed to be an insulator, i.e. if there is no clay, the imaginary part of the dielectric constant is negligible and the CRIM relation becomes

$$\Phi = \frac{\sqrt{k_{total}} - \sqrt{k_{sand}}}{\sqrt{k_{air}} - \sqrt{k_{sand}} + S_u \left( \sqrt{k_{water}} - \sqrt{k_{air}} \right)} \quad (14)$$

where $k$ is the dielectric constant of the medium ($\approx 4.8$ for the sand, 1 for air and 80 for water; Knight and Endres, 1990), $S_u$ is the saturation coefficient (between 0, dry, and 1, saturated) and $\Phi$ is the total porosity.

In order to compare the real stratigraphy of the test site with our results, two vertical holes were bored using a hand auger a few days before the GPR survey (Fig. 2). Sand samples were collected each 0.3 m between 0 and 2.5 m. From the analysis of the collected sand samples, it was observed that the volume of clay is negligible. Therefore, the porosity can be estimated using Eq. (14). The hydraulic con-
ductivity can be estimated using the Kozeny–Carman relation (Todd, 1980, p. 361):

\[ K = \frac{C g \Phi^3}{\mu_{\text{water}} \rho_{\text{water}} S_p^2 D_R^2 (1 - \Phi)^2} \]  

(15)

where \( K \) is the hydraulic conductivity (m s\(^{-1}\)), \( C \) is a constant related to the shape of the grain, \( g \) is the gravitational acceleration (9.8 m s\(^{-2}\)), \( \mu_{\text{water}} \) is the dynamic viscosity of water (1.2 \times 10^{-3} \text{ Pa s} ), \( \rho_{\text{water}} \) is the volumetric mass of water (kg m\(^{-3}\)), \( S_p \) is specific surface of sand (m\(^2\) kg\(^{-1}\)) and \( \Phi \) is the total porosity.

The greatest difficulty with the Kozeny–Carman relation is to determine the specific surface. One method uses the granulometric curve (Chapuis and Montour, 1992; Chapuis and Légaré, 1992). Since the granulometric curves are not known at the interpolation grid, we have to assume that this parameter is constant throughout the field. The impact of this assumption will be to decrease somehow the variance of the estimated hydraulic conductivities and to decrease the precision of the estimates.

For a given porous medium, there exists a critical porosity, \( \Phi_c \), above which, water can flow freely. From Mavko and Nur (1997), the threshold for sands is about 2–4%. The Kozeny–Carman relation can be adapted to take it into account

\[ K = \frac{C g (\Phi_c - \Phi)^3}{\mu_{\text{water}} \rho_{\text{water}} S_p^2 D_R^2 (1 - \Phi - \Phi_c)^2} \]  

(16)

The results of 206 experiments show an average \( C \) value of about 0.39 for sand (Chapuis and Montour, 1992). Laboratory measurements indicate \( \mu_{\text{water}} = 0.0012 \text{ Pa s} \) and \( \rho_{\text{water}} = 999.84 \text{ kg m}^{-3} \) for water at 25 \(^\circ\)C. The variation of these parameters with temperature is too small to have a significant influence on the results.

2.4. Estimation of water content

This section shows how to compute the water content and the porosity using Topp relation.

Topp et al. (1980) have shown that the dielectric constant of a granular mineral soil (varying from 3 to 40) is strongly dependent on the water content (changing from 0% to 0.55%) and only weakly dependent on the actual porosity, soil type, density and temperature for frequencies between 20 MHz and 1 GHz. They propose a simple empirical relation between the volumetric water content and the dielectric constant of a soil

\[ V_w = -5.3 \times 10^{-2} + 2.92 \times 10^{-2} k_{\text{total}} - 5.5 \times 10^{-4} k_{\text{total}}^3 + 4.3 \times 10^{-6} k_{\text{total}}^3. \]  

(17)

Other relations have been derived to estimate volumetric water content from the bulk dielectric constant (Dannowski and Yamaranci, 1999; Yu et al., 1999). They all give estimates very close to the ones obtained using Topp and CRIM relations.

The volumetric water content, \( V_w \), is simply related to saturation and porosity (Freeze and Cherry, 1979, p. 34).

\[ V_w = S_w \Phi. \]  

(18)

For the saturated layer, the volumetric water content is equivalent to the porosity, because saturation is assumed to be 100%. Then, the porosity estimated with Topp relation can be compared with the estimated porosity using CRIM relation. Topp and CRIM relations yield nearly identical porosities in the saturated zone (correlation between both estimates is 0.9997).

However, in the unsaturated layer, it is not possible using Eq. (14) to estimate directly the spatial distribution of the porosity because water saturation is unknown. Thus, we can use Eq. (17) to compute \( V_w \). Then, combining Eqs. (14), (17) and (18), the porosity can be expressed as a function of \( V_w \)

\[ \Phi = \frac{\sqrt{k_{\text{total}}} - \sqrt{k_{\text{sand}}} + V_w (\sqrt{k_{\text{air}}} - \sqrt{k_{\text{water}}})}{\sqrt{k_{\text{air}}} - \sqrt{k_{\text{sand}}}} \]  

(19)

which enables to determine the porosity of the unsaturated zone.

3. Acquisition and analysis

3.1. Acquisition

In December 1998, a GPR survey was performed at the test site of Ecole Polytechnique, located in
Lachenaie, near Montreal (Canada). The shallow unconfined aquifer consists of a 3.5-m thick sand layer with some thin intercalated silt layers overlying a 20-m thick Champlain clay layer (Fig. 1).

The grain size of the sand is medium-to-coarse ($D_{50} = 0.28$ mm and $D_{60} = 0.71$ mm) and the clay is an aquitard (with hydraulic conductivity of $10^{-7}$ cm s$^{-1}$). The GPR survey area is $6.25 \times 5$ m. Seven piezometers, distributed in a cross-like manner (Fig. 2), allow collection of the hydrogeological measurements. Depth-to-clay was measured during the installation of the piezometers and the hydraulic head was measured a few days before the GPR survey. Seven depth-to-clay data and five hydraulic head data were available. A PulseEkko IV (Sensors and Softwares) with a 200-MHz antenna was used for collecting the GPR data. The sampling interval is 0.2 m along the lines spaced by 0.25 m. The antennae were perpendicular broadside-oriented and 64 traces were stacked at each point.

The GPR traces were band-pass-filtered, corrected for drift and static corrections have been applied. In Fig. 3, a 3D cube has been generated using all the traces. Amplitudes are gained using a SEC gain and are displayed as function of position $(x, y)$ and time. A Spherical Exponential Compensation (SEC) gain consists of choosing the best velocity and attenuation of the ground in order to most effectively compensate for spherical spreading and exponential attenuation.

### 3.2. Analysis

The radar reflection at the saturated–unsaturated sand interface does not occur at the piezometric level, but at the maximum gradient of the capillary fringe. From the in situ volumetric water content measurements, we found a capillary rise, $h_c$, of approximately 30 cm above the piezometric level. This value is a common value for medium-to-coarse sands (Todd, 1980, p. 35). The maximum capillary gradient is at mid-height of the capillary fringe (Fig. 1). To compute the velocity, the first half of the capillary fringe is included in the aeration zone, and the other half is included in the saturated zone. To ensure that the good reflectors were picked, we modeled the GPR response for the hydrostratigraphic
model of the survey zone using pulseEKKO42 synthetic radargram software (Fig. 3). The model consists of an unsaturated sand layer of 1.6 m with a dielectric constant of 12. The capillary fringe is located between 1.6 and 1.9 m and the dielectric constant increases from 12 to 22, following the capillary fringe model (Todd, 1980, p. 32). From the 1D synthetic radar traces, we found that the saturated–unsaturated sand interface (USI) reflection occurs at 40 ns and the sand–clay interface (SCI) reflection occurs at 100 ns. From the in situ sand samples, we correlated the reflection at 20 ns with a thin layer of high water content (rising rapidly from 5–10% to 30%). The reflections between 50 and 100 ns are possibly caused by slight variations in the sand granulometry (silty, coarse) or by multiples reflections of the upper reflectors.

Using the results of the 1D synthetic, the reflection times corresponding to the USI and SCI were systematically picked for all traces. Times were corrected for the offset delay. The offset delay is the time taken by the radar wave to travel from the transmitter to the receiver in the air. Reflection times were then correlated with piezometric information, piezometric level and depth-to-clay, where available.

Simple correlations between the measured depths to the water table and clay layer and radar travel time measurements were calculated to be 0.9 and 0.8 for USI and SCI, respectively. The computed experimental variograms of reflection times for both interfaces did not show anisotropy. Therefore, Figs. 4 and 5 show only the omnidirectional variograms. Both experimental variograms were modeled. For the USI reflection time, the model is spherical with a sill of 2.1 ns² and a range of 1.5 m and, for the SCI reflection time, the model is exponential with a sill of 7.5 ns² and a range of 0.6 m.

The cross-validation is computed with the program COKRI developed by Marcotte (1991, 1993). The difference between cross-validated and measured data appears to be Gaussian. For both variables, we calculated the mean squared standardized errors, which consist in the square difference between observed and calculated data divided by the cokriging variances. These statistics should be close to one if the cokriging variances adequately predict the precision of the estimation. We obtained 0.9936 for travel time in water and 0.9523 for travel time in clay (measured at 676 points) and 1.08 for water depth (measured at five points) and 1.06 for clay depth (measured at seven points). Fig. 6 displays the depth distribution obtained by cokriging. The mean depth of the water table is 1.9 m and the mean depth to the clay is 3.5 m. The variations of the water table are centimetric, whereas the variations are decimetric for the clay depth.

Because the ordinary cokriging estimator has no bias, the estimated reflectors depth is only defined

Fig. 3. 3D representation of the GPR data with synthetic radar trace. DA: Direct arrivals of radar wave in the air; Rf: reflection on a high water content layer; CFG: capillary fringe gradient; Clay: clay surface.
Fig. 4. Theoretical and experimental omnidirectional variogram for EM travel time from surface to water table (spherical model of range, 1.5 and sill, 2.1).

Fig. 5. Theoretical and experimental omnidirectional variogram for EM travel time from surface to clay surface (exponential model of range, 0.6 and sill, 7.5).
with the reflection time structure and the piezometrical measurements. This means that if there is a constant translation of all the reflection times, the estimated depth will not vary. We know that the reflected wave is not exactly vertical because of the offset distance between the transmitter and the receiver. The time difference between an ideally vertical reflected wave (zero-offset) and the actual finite-offset two-way travel time is called the offset time. At the points where we measured the piezometrical data and the travel time, the offset correction is simply a geometric correction. Because we know that the spatial variations of physical parameters (electrical conductivity) are small, we made the offset correction in the same manner with the estimated depth and the measured travel times. If there were large spatial variations of the physical properties, depths to the reflectors would have to be known in every place where the changes occur.

Using the estimated depths for the key horizons and the travel times for both reflectors, we computed the velocity of the two layers (Figs. 7 and 8). The geometric mean of the estimated velocities for the unsaturated sand is about 0.088 m ns\(^{-1}\) with a standard deviation of 0.0032 m ns\(^{-1}\), whereas for the saturated sand, the velocity is about 0.064 m ns\(^{-1}\) with a standard deviation of 0.0022 m ns\(^{-1}\). We also did two CMP surveys located at 70–29 and 70–32. We ran a velocity analysis using the pulseEKKO42 software. The mean velocity for the first layer was 0.10 ± 0.01 m ns\(^{-1}\). The saturated layer has a mean interval velocity of 0.069 ± 0.01 m ns\(^{-1}\). Thus, in average, the two methods give close results. One advantage of using surface GPR reflection times over CMP approach is that with the former, we can assess the spatial distribution of the velocity. Also, with GPR reflection data, each value is associated with a small volume of sand, whereas the CMP velocity is an integration of the velocities over a large volume of sand.

The usual perfectly dry sand velocity is around 0.15 m ns\(^{-1}\) (Annan and Daniels, 1998), whereas our estimated velocities of the aeration zone have a geometric mean of 0.088 m ns\(^{-1}\). This difference can be explained by the high water content in pores. Indeed, the dielectric constant of water is much larger than the dielectric constant of the dry sand (80 for water and 4.7 for dry sand) and a small amount of water in pores increases dramatically the dielectric constant of the medium. We note that we had 7
mm/day of rain during the 10 days preceding the survey. Also, we note that the 20-ns reflection (Fig. 3) is possibly a retention layer located approximately at 1 m under the surface. In the in situ samples, we found volumetric water content higher than 30% in this layer.
Using the estimated velocity in Eq. (8), we computed the spatial distribution of the dielectric constants of both layers. Fig. 9 shows the results, which agree with the published values (Annan and Daniels, 1998). The geometric mean is 11.7 for the unsaturated sand and 22 for saturated sand. The standard deviations are 0.85 and 1.53, for the unsaturated and saturated sand, respectively. For the unsaturated layer, the values are larger than expected, but again, it can be explained by the high water content in the pores.

Generally, the dielectric constants of air and water are almost invariant over a survey area. Moreover, under the assumptions that the CRIM and the Topp relations give similar estimates of the water content and that the sand is completely saturated in the saturated zone \( S_s = 1 \), it is possible to compute the dielectric constant of the pure sand. Using this approach with Eqs. (14), (17) and (18), the dielectric constant of the Lachenaie sand is found to be 4.85.

Under the assumption that the dielectric constants of the sand and of the water are invariant over the survey area and using Eqs. (14), (17) and (18), we estimated the porosity of the saturated sand. The geometric mean of the calculated porosity using both methods is \( 37 \pm 0.05\% \) which compares well with the measured value on sand samples in laboratory (40%). Fig. 10 shows the inferred porosity distribution. Significant spatial variation in the estimated porosity is visible, but the mean value obtained is comparable with the value obtained in laboratory.

Using the porosity estimated with the CRIM formulation, the Kozeny–Carman relation is used to estimate the distribution of the hydraulic conductivity (Fig. 11). The mean of the two laboratory measurements of the specific surface (6.68 and 7.24 m\(^2\) kg\(^{-1}\) from Chapuis et al., 1998) was used to compute the modified Kozeny–Carman relation (Eq. (16)). The critical porosity was taken to be 3\%, the mean value for sands as proposed by Mavko and Nur (1997) and Nur et al. (1998). The geometric mean of the estimated hydraulic conductivity is \( 8 \times 10^{-4} \) m/s whereas the pumping and tracer tests give a mean conductivity of \( 7.8 \times 10^{-4} \) m/s with a standard deviation of \( 1.1 \times 10^{-4} \) m/s (Chapuis et al., 1997; Chapuis et al., 1998). The standard deviation of our estimate is \( 2.2 \times 10^{-4} \) m/s. The geometric mean of the estimated conductivity is close to the value measured during the pumping test despite the fact that we had to assume that some parameters such as the

![Dielectric constant distribution in the unsaturated and saturated layers](image-url)
Fig. 10. Estimated porosity distributions in the saturated zone using both CRIM and Topp relations (white circles indicate where stratigraphic information was available).

Fig. 11. Distribution of estimated hydraulic conductivity in the saturated sand (white circles indicate where stratigraphic information was available).
specific surface are constants over the volume of the sand investigated.

The horizontal distribution of the soil moisture in the unsaturated sand was estimated using Topp relation. The geometric mean of the volumetric water content is about 22%, the minimum 18% and the maximum 24%. Estimates obtained from GPR data can be compared with the moisture content measured at the lab in sand samples using an oven-drying technique (Freeze and Cherry, 1979, pp. 213–216). These estimated parameters represent an average of the vertical distribution of the soil moisture between the surface and the USI interface. These values are larger than those obtained with the in situ samples (15%). However, the soil sampling was done 10 days before the GPR survey. As it rained steadily during the period between the sand sampling and the GPR survey, it is expected that the estimated moisture should be higher than the one obtained in the sand sample analysis.

Knowing the dielectric constants of the materials (Fig. 12) and using the volumetric water content relation (Eq. (17)), the porosity within the unsaturated layer can be estimated by Eq. (19). The average porosity for the unsaturated zone is 44% with a standard deviation of 0.0017. This compares well with the porosity measured in the laboratory (40%).

In addition to using GPR travel time information, hydrogeological information is also potentially available from GPR signal amplitude measurements. The attenuation coefficients have been estimated from some of the collected radar traces for the unsaturated and saturated zones. The attenuation was calculated for the unsaturated zone using three techniques. First, it was computed from the amplitude envelope with time (using the EKKO42 software subroutine POWRLOT). This technique is based on the instantaneous attributes obtained by taking the Hilbert transform of the traces (Yilmaz, 1988, p. 521). Secondly, a SEC gain (see Section 3.1) was applied to traces until the reflection amplitude was similar to the direct ground arrival amplitude. Third, the attenuation was estimated from the attenuation of amplitude of the direct arrivals in the ground from CMP measurements using Eq. (5). For the saturated layer, only the first two techniques were applied because the direct arrival in the ground only goes through the first layer. The mean of the estimated attenuation is

![Fig. 12. Estimated porosity of the aeration zone using Topp and CRIM relations (white circles indicate where stratigraphic information was available).](image-url)
0.46 Np/m for the unsaturated sand and is 1.05 Np/m for the saturated sand. The bulk conductivity of the ground was then estimated using the relation between the observed attenuation coefficient, the previously estimated dielectric constant and the bulk conductivity (Eq. 6). Partitioning of the energy at the interfaces is included in the observed attenuation when using reflected waves. The geometric mean of the estimated electric conductivity is 25 mS/m for the saturated layer. Recently, a 3D pole–pole resistivity tomography survey was performed on the radar grid. Modeling show electric conductivities about 20–33 and 1–2.5 mS/m for the saturated and unsaturated layers, respectively, which agree well with the conductivity estimate computed from radar attenuation.

It is possible to estimate the porosity from the electric conductivity of the saturated medium if the electric conductivity of the pore water is known. Archie’s formulation states that (Annan and Daniels, 1998)

\[
\sigma = \sigma_w \epsilon_w \sigma_{\epsilon} + \sigma_c
\]  

(20)

where \(\sigma_w\) is the electric conductivity of the water and \(\sigma_c\) is the electric conductivity of the soil grain surface and \(a, m\) and \(n\) are constants (Annan and Daniels, 1998; Reynolds, 1997, p. 348). From the literature, we choose, as appropriate constants, values for \(a, m\) and \(n\) are 0.88, 1.3 and 2, respectively. Because of the low electric conductivity of the grain matrix with regard to the water conductivity, \(\sigma_c\) can be neglected. The water electric conductivity was found to be constant in the five piezometers. Its value is 105 mS/m at \(T = 10^\circ\)C. Using Eq. (20), the porosity was estimated to be 36%, which agrees well with GPR estimate (37%). For the unsaturated layer, the estimate is dependent on the electric conductivity of the water which may well be lower in that zone since it consists of water percolating from recent rain falls. We have no electric conductivity measurements available of the water in the unsaturated zone. Using bulk conductivities of 2 mS/m obtained from the DC resistivity survey and the volumetric water content determined previously from radar velocities, electric conductivity of the water within the unsaturated zone is estimated to be 20 mS/m.

3.3. Error estimation

In this section, we list possible errors and study their effect on our analysis.

We stated that the topography was flat. In fact, a high-resolution topographic survey of the area has shown that the maximum elevation difference is 0.06 m. This mean an error of \(6 \times 10^{-1}\) ns, for a average velocity of 0.09 m ns\(^{-1}\). Thus, the topography can be neglected for this survey site in comparison with the trace sampling interval of 0.8 ns.

The error on specific surface measurements is, at least, equal to the difference between the two measured values, 0.56 m\(^2\) kg\(^{-1}\). The critical porosity is known within a range of 3%. However, those parameters were considered constants, an assumption that reduces the estimated variances.

The cross-validation gives an error about 2 ns and 2 cm for dry sand and 5 ns and 7 cm for saturated sand. As we have shown, the mean square standardized error confirms that the chosen model is adequate.

The error on each picking is about ±1 sample interval, 0.8 ns. Thus, the maximum error on the picking of the saturated–unsaturated interface is about 1.6 ns and about 2.4 ns for the sand–clay interface (the picking error is summed at each interface).

A white noise of maximum 3.5 ns and standard deviation of 2 ns which represent well the sum of the above described noises were added to the picked times. The covariogram models were modified to account for the new perturbed data and the porosity and the hydraulic conductivity were computed again. The geometric means of the porosity and of the conductivity with and without the noise are within 1% of each other; the spatial distribution remains the same.

Because ordinary cokriging is independent of the means of the variables, the estimated porosity and hydraulic conductivity structures do not change if a constant time delay is added to the GPR travel times.

4. Discussion

In the cokriging, there is no limit on the number of variables that can be used. Thus, any other geo-
physical method, which gives information on subsurface topography of the interfaces, can be used as a secondary variable. For example, seismic refraction or resistivity survey may give important information on subsurface layers depth.

The applicability of the presented method to more complex aquifers has been tested on synthetic models (Gloaguen et al., 2000). If the topography of the interfaces between the layers is complex but the properties within the layers are homogeneous, cokriging provides good determination of both layer depths and subsequently, of the dielectric constants. However, variations on both layer topography and physical properties bring a greater uncertainty on the estimation of the parameters. Information from additional piezometers or indirect determination of dielectric constants by TDR or borehole radar survey would have to be used to reduce the indeterminacy in the problematic areas.

In many cases, only a few numbers of piezometers are available. Fig. 13 shows the results of the computed hydraulic conductivity using only three piezometers. Their positions are 70–30, 70–32.5 and 69–30. The geometric mean is $8.0 \times 10^{-4} \text{ m/s}$ with a standard deviation of $1.84 \times 10^{-4} \text{ m/s}$. The results are very similar to the one obtained using all the piezometers ($8 \times 10^{-4} \text{ m/s}$). The standard deviation is slightly lower with three piezometers than with all the piezometers as expected because the cokriged interfaces are also smoothed with a smaller number of data.

The dielectric constant and attenuation fields estimated from the GPR surface data could also be either validated or constrained using transmission radar measurements with small borehole probes inserted in the piezometers. The moisture content in the unsaturated zone could also be determined using moisture probes.

The spread of the statistical distribution of estimated parameters is probably a combined effect of the real spatial variability of these parameters and the consequence of the processing errors. At this stage, it is not possible to determine separately the contributions of these two factors.

Many of the parameters used in this study had to be fixed more or less arbitrarily sometimes because we had no measurements of the parameter (dielectric constants of sand and water) and sometimes, because

Fig. 13. Distribution of estimated hydraulic conductivity in the saturated sand with only three piezometers (whites circles) located at 70–30, 70–32.5 and 69–30.
we had not enough data to have an idea of the spatial distribution of the parameter (specific surface measurements). Also, the estimated porosities represent an average value along a vertical column between two interfaces. Thus, within each layer, only lateral variations were considered in our model.

One key variable in the velocity determination is the volumetric water content in the unsaturated zone. This quantity is highly sensitive to climatic conditions prevailing a few days before the survey. Moreover, these conditions could be substantially different from those prevailing at the time when sand samples were collected in the field. Therefore, comparison of measured properties with those estimated from cokriging GPR data is rather difficult. As a rule, hydrogeological and geophysical data should be collected simultaneously.

5. Conclusion

In spite of the limitations presented above, we have shown that it is possible to use well-sampled geophysical data to complete under-sampled hydraulic head or stratigraphic data. The small area size of the test site does not limit the applicability of the method to larger areas.

We have shown that it is possible to have an estimation of soil porosity and hydraulic conductivity using geophysical and hydrogeological data and geostatistics tools. In this respect, one important contribution of GPR data is to enable to model the covariance function for the “depth-to-interface” variable. This would not have been otherwise possible using only seven piezometrical values. A precise mapping of the position of the interfaces was obtained with a small number of piezometers which could result in savings.

In addition to the “depth-to-interface” estimation, GPR enables mapping of the lateral variations of porosity and then, of hydraulic conductivity, assuming a know mean specific surface value. Note that the relative values of these parameters between adjacent grid positions are not very sensitive to the various assumptions made for their computation; however, their absolute values are. The piezometric head is locally a function of the contrast of conductivities. Thus, the natural flow path is expected to be well-described by the estimated hydraulic conductivity field in the saturated zone.

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